

# **GRADE 8**

### Open-Ended Problem-Solving Projections

Organized by TEKS Categories

### **Mathematical Process Standards**

These student expectations will not be listed separately. They will be incorporated into assessments for TEKS in other categories since the application of mathematical process standards is part of each knowledge statement for all other TEKS.

#### 8.(1) Mathematical Process Standards

The student uses mathematical processes to acquire and demonstrate mathematical understanding.

TEKS	STUDENT EXPECTATION
8.1(A)	apply mathematics to problems arising in everyday life, society, and the workplace
8.1(B)	use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution
8.1(C)	select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems
8.1(D)	communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate
8.1(E)	create and use representations to organize, record, and communicate mathematical ideas
8.1(F)	analyze mathematical relationships to connect and communicate mathematical ideas
8.1(G)	display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication

### **Number and Operations**

### 8.(2) Number and Operations

The student applies mathematical process standards to represent and use rational numbers in a variety of forms.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.2(A)	extend previous knowledge of sets and subsets using a visual representation
		to describe relationships between sets of real numbers
Supporting	8.2(B)	approximate the value of an irrational number, including $\pi$ and square roots
		of numbers less than 225, and locate that rational number approximation on
		a number line
Supporting	8.2(C)	convert between standard decimal notation and scientific notation
Readiness	8.2(D)	order a set of real numbers arising from mathematical and real-world
		contexts

### Proportionality

### 8.(3) Proportionality

The student applies mathematical process standards to use proportional relations to describe dilations.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.3(A)	generalize that the ratio of corresponding sides of similar shapes are
		proportional, including a shape and its dilation
Supporting	8.3(B)	compare and contrast the attributes of a shape and its dilations(s) on a
		coordinate plane
Readiness	8.3(C)	use an algebraic representation to explain the effect of a given positive rational scale factor applied to two-dimensional figures on a coordinate plane with the origin as the center of dilation

### Proportionality

### 8.(4) Proportionality

The student applies mathematical process standards to explain proportional and nonproportional relationships involving slope.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.4(A)	use similar right triangles to develop an understanding that slope, $m$ , given as the rate comparing the change in y-values to the change in x-values,
		$(y_2 - y_1)/(x_2 - x_1)$ , is the same for any two points $(x_1, y_1)$ and $(x_2, y_2)$ on the same line
Readiness	8.4(B)	graph proportional relationships, interpreting the unit rate as the slope of the line that models the relationship
Readiness	8.4(C)	use data from a table or graph to determine the rate of change or slope and y-intercept in mathematical and real-world problems

### Proportionality

### 8.(5) Proportionality

The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.5(A)	represent linear proportional situations with tables, graphs, and equation in the form of $y = kx$ .
Supporting	8.5(B)	represent linear non-proportional situation with tables, graphs, and equations in the form of $y = mx + b$ , where $b \neq 0$
Supporting	8.5(C)	contrast bivariate sets of data that suggest a linear relationship with bivariate sets of data that do not suggest a linear relationship from a graphical representation
Readiness	8.5(D)	use a trend line that approximates the linear relationship between bivariate sets of data to make predictions
Supporting	8.5(E)	solve problems using direct variation

### Proportionality

### 8.(5) Proportionality

The student applies mathematical process standards to use proportional and non-proportional relationships to develop foundational concepts of functions.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.5(F)	distinguish between proportional and non-proportional situations using tables,
		graphs, and equations in the form of $y = kx$ or $y = mx + b$ , where $b \neq 0$
Readiness	8.5(G)	identify functions using sets of ordered pairs, tables, mappings, and graphs
Supporting	8.5(H)	identify examples of proportional and non-proportional functions that arise
		from mathematical and real-world problems
Readiness	8.5(I)	write an equation in the form $y = mx + b$ to model a linear relationship
		between verbal, numerical, tabular, and graphical representations

### **Expressions, Equations, and Relationships**

#### 8.(6) Expressions, Equations, and Relationships

The student applies mathematical process standards to develop mathematical relationships and make connections to geometric formulas.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.6(A)	describe the volume formula $V = Bh$ of a cylinder in terms of its base area
		and its height
Not tested	8.6(B)	Model the relationship between the volume of a cylinder and a cone having
		both congruent bases and heights and connect that relationship to the
		formulas
Supporting	8.6(C)	use models and diagrams to explain the Pythagorean Theorem

### **Expressions, Equations, and Relationships**

#### 8.(7) Expressions, Equations, and Relationships

The student applies mathematical process standards to use geometry to solve problems.

STAAR Standard	TEKS	STUDENT EXPECTATION
Readiness	8.7(A)	solve problems involving the volume of cylinders, cones, and spheres
Readiness	8.7(B)	use previous knowledge of surface area to make connections to the formula
		for lateral and total surface area and determine solutions for problems
		involving rectangular prisms, triangular prisms and cylinders
Readiness	8.7(C)	use the Pythagorean Theorem and its converse to solve problems
Supporting	8.7(D)	determine the distance between two points on a coordinate plane using the
		Pythagorean Theorem

### **Expressions, Equations, and Relationships**

#### 8.(8) Expressions, Equations, and Relationships

The student applies mathematical process standards to use one-variable equations or inequalities in problem situations.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.8(A)	write one-variable equations or inequalities with variables on both sides that
Supporting	8.8(B)	write a real-world problem when given a one-variable equation or inequality with variables on both sides of the equal sign using rational number coefficients and constants
Readiness	8.8(C)	model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants
Supporting	8.8(D)	use informal arguments to establish facts about the angle sum and exterior angle of triangles, the angles created when parallel lines are cut by a transversal and the angle-angle criterion for similarity of triangles

	Expressions, Equations, and Relationships			
8.(9) Expressions, Equations, and Relationships The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations.				
STAAR Standard	TEKS	STUDENT EXPECTATION		
Supporting	8.9(A)	identify and verify the values of x and y that simultaneously satisfy two linear equations in the form $y = mx + b$ from the intersections of the graphed equations		

### **Two-Dimensional Shapes**

### 8.(10) Two-Dimensional Shapes

The student applies mathematical process standards to develop transformational geometry concepts.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.10(A)	generalize the properties of orientation and congruence of rotations, reflections, translations, and dilations of two-dimensional shapes on a coordinate plane
Supporting	8.10(B)	differentiate between transformations that preserve congruence and those that do not
Readiness	8.10(C)	explain the effects translations, reflections over the $x$ - or $y$ -axis, and rotations limited to 90°, 180°, 270°, and 360° as applied to two-dimensional shapes on a coordinate plane using an algebraic representation
Supporting	8.10(D)	model the effect on linear and area measurements of dilated two-dimensional shapes

### **Measurement and Data**

#### 8.(11) Measurement and Data

The student applies mathematical process standards to use statistical procedures to describe data.

STAAR Standard	TEKS	STUDENT EXPECTATION
Supporting	8.11(A)	construct a scatterplot and describe the observed data to address questions of association such as linear, non-linear, and no association between
		bivariate data
Supporting	8.11(B)	determine the mean absolute deviation and use this quantity as a measure
		of the average distance data are from the mean using a data set of no more
		than 10 data points
Not Tested	8.11(C)	Simulate generating random samples of the same size from a populations
		with known characteristics to develop the notion of a random sample being
		representative of the populations from which is was selected

### **Personal Financial Literacy**

### 8.(14) Personal Financial Literacy

The student applies mathematical processes standards to develop an economic way of thinking and problem solving useful in one's life as a knowledgeable consumer and investor.

STAAR Standard	TEKS	STUDENT EXPECTATION				
Supporting	8.12(A)	solve real-world problems comparing how interest rate and loan length				
		affect the cost of credit				
Not Tested	8.12(B)	Calculate the total cost of repaying a loan, including credit cards and				
		easy access loans, under various rates of interest and over different				
		periods using an online calculator				
Supporting	8.12(C)	explain and compare simple interest and compound interest earnings				
Readiness	8.12(D)	Calculate and compare simple interest and compound interest earnings				
Not Tested	8.12(E)	Identify and explain the advantages and disadvantages of different				
		payment methods				
Not Tested	8.12(F)	Analyze situations to determine if they represent financially responsible				
		decisions and identify the benefits of financial responsibility and the costs				
		of financial irresponsibility				
Supporting	8.12(G)	estimate the cost of a two-year and four-year college education,				
		including family contribution, and devise a periodic savings plan for				
		accumulating the money needed to contribute to the total cost of				
		attendance for at least the first year of college				

### **GRADE 8 PROJECTION MASTERS for PROBLEM-SOLVING**

### **OVERVIEW**

The Projection Masters were created with all students in mind and provide teachers with large print projections for problem-solving questions that address all TEKS. Each Projection Master is correlated to a specific Category and TEKS.

The Projection Masters document includes a general set of questions that should be addressed by students as they solve the problems and during class discussions of the solution process for each problem. Teachers should make a copy of these questions and distribute for each student to keep in his/her math notebook.

#### The Problem-Solving Questions include the following:

- 1. What is the main idea of this problem?
- 2. What are the supporting details in this problem?
- 3. What skills, concepts, and understanding of math vocabulary are needed to be able to answer this question?
- 4. Did this problem involve mathematics arising in everyday life, society, or the work place?
- 5. What is a good problem-solving strategy for this problem?
- 6. Can you explain how you used any math tools, mental math, estimation, or number sense to solve this problem?
- 7. Did this problem involve using multiple representations (symbols, diagrams, graphs, language)?
- 8. Did you use any relationships to10 solve this problem?
- 9. How can you justify your solution?
- 10. How can you check for reasonableness of your solution to this problem?

These Projection masters can be utilized for instruction, guided practice or independent practice. These materials can be utilized with a whole class, small groups, and/or tutorial settings.

**NOTE:** There is no answer key provided fro the Projection Masters as the authors' philosophy is that each teacher should create a personalized Solutions Manual so the teacher becomes more familiar with the Revised TEKS and assessment of the Revised TEKS, as well as formulates various solution strategies for each question. Teachers are encouraged to communicate with the authors regarding discussion of any questioning this document.

### AUTHORS' VISION FOR IMPLEMENTATION

Students work with a partner pair to answer the open-ended problems. The teacher projects the problem then sets a time limit prior to students beginning their work. Partner pairs are given specific "share" questions from the 1-10 on the Problem-solving Questions page. The process should be followed by students for all open-ended questions is to answer questions 1-3, then complete the solution to the problem, and then finally answer questions 4-10.

The teacher calls time and the partner pairs guide the class discussion on their "share" assignments, Students who did not complete the solution to the problem prior to the time limit must complete recording in a different color.



### **TEKS CATEGORY 1** Numbers and Operations

### **Problem-Solving Questions**

### **Directions:**

- Work with a partner.
- Write your answers on notebook paper.
- Answer questions 1-3.
- Complete the solution to the problem(s).
- Answer questions 4-10.
- 1. What is the main idea of this problem?
- 2. What are the supporting details in this problem?
- 3. What skills, concepts, and understanding of math vocabulary are needed to be able to answer this problem?
- 4. Did this problem involve mathematics arising in everyday life, society, or the work place?
- 5. What is a good problem solving strategy for this problem?
- 6. Can you explain how you used any math tools, mental math, estimation, or number sense to solve this problem?
- 7. Did this problem involve using multiple representations (symbols, diagrams, graphs, math language)?
- 8. Did you use any relationships to solve this problem?
- 9. How can you justify your solution to the problem?
- 10. How can you check for reasonableness of your solution to this problem?



## **TEKS CATEGORY 1** 8.2A

### 8.2A Problem Solving 1

Draw a Venn diagram to show the relationship between the factors of 20 and 75. Be sure to label your diagram.

Factors of 20: \_\_\_\_\_ Factors of 75: \_\_\_\_\_

What factors do they have in common?

Where do they appear in the diagram?

### 8.2A Problem Solving 2

**Problem 1:** Give an example of a situation where the values that make sense are only the given subset of real numbers.

- Even positive integers
- Positive rational numbers
- Rational numbers

### **Problem 2:** Place a $\sqrt{}$ in each column that the given number belongs to.

	Real Number	Irrational Number	Rational Number	Integer	Whole Number	Counting Number
-4						
2π						
3.5						

### 8.2A Problem Solving 3

**Problem 1:** In your own words, describe how to determine if a decimal number is a rational number or an irrational number.

**Problem 2:** Circle the decimals in the list below that are rational decimals.

- 3.1253.11111...1.21121112...
- 2.6 0.01020304...

**Problem 3:** Express each of the following decimals as a ratio of two integers.

3.75 4.22222...



### **TEKS CATEGORY 2** Proportionality



### **TEKS CATEGORY 2** 8.5E

### 8.5E Problem Solving 1

**Problem 1**: The cost of a soft drink varies directly with the number of ounces you buy. If 15 ounces cost \$0.60, what would you pay for 32 ounces of the soft drink? Show your work.

**Problem 2:** The number of gallons of gasoline used is directly proportional with the number of miles traveled. If traveling 240 miles requires 16 gallons, how many gallons would a 600-mile trip require? Show your work.

**Problem 3:** If y is directly proportional with x and y = 20 when x = 8, what is the value of y when x = 24? Show your work.

### 8.5E Problem Solving 2

**Problem 1**: The cost of gasoline varies directly with the number of gallons you buy. If 3 gallons cost \$9.30, what would you pay for 20 gallons of gasoline? Show your work.

**Problem 2:** The number of words typed varies directly with the number of minutes typed. If Sue can type 60 words in 1.5 minutes, how many words can she type in 20 minutes? Show your work.

**Problem 3:** If y is directly proportional with x and y = 45 when x = 3, what is the value of y when x = 12? Show your work.



### **TEKS CATEGORY 2** 8.5G

### 8.5G Problem Solving 1

### **Problem 1:**

- What must be true about a set of ordered pairs if they describe a function?
- Circle the sets of ordered pairs below that describe a function.

 $\{(1, 2), (4, 5), (0, 1), (5, 6)\}$   $\{(3, 2), (4, 4), (1, 4), (3, 5)\}$ 

 $\{(1, 2), (1, 5), (1, 1), (1, 6)\}$   $\{(2, 2), (4, 4), (1, 1), (5, 5)\}$ 

**Problem 2:** How do you look at a graph and decide if it describes a function?

**Problem 3:** Does a graph have to be a line to represent a function? Explain.

### 8.5G Problem Solving 2

### Problem 1:

• How do you decide if a mapping is a function?

Create a mapping for the two sets below so that it is a function.



### Problem 2:

- How do you decide if a table of values describes a function?
- The table of values below is NOT a function. Why?

X	2	3.5	2	5	2	6	1
У	5	6	9	6	4	13	9

### 8.5G Problem Solving 3

Identify the following 4 graphs as a function or NOT a function.







### **TEKS CATEGORY 3** Expressions, Equations and Relationships



### **TEKS CATEGORY 3** 8.7A

### 8.7A Problem Solving 1

**Problem 1:** A cone has a radius length that is half the height of the cone.



- Write the formula from your Resource Materials that can be used to find the volume of the cone.
- Write a formula in terms of *h* that can be used to find the volume of the cone.
- Use the formula to find the volume of the cone if the height is 12 inches. Show your work.

**Problem 2:** A cone has a base area of  $25\pi$  square units and a volume of  $300\pi$  cubic units.

- Draw and label a sketch of the cone.
- Find the height of the cone. Show your work.

### 8.7A Problem Solving 2

**Problem 1:** A sphere has a radius of *r* meters.



- •Record the formula for the volume of a sphere.
- •Use the formula to calculate the volume for three different spheres with radius lengths of 3, 6, and 9 units. Show your work.

**Problem 2:** Look at the volumes you found for the spheres with radius lengths of 3, 6, and 9 units in Problem 1.

How did the volume change as the radius doubled from 3 units to 6 units?

How did the volume change as the radius tripled from 3 units to 9 units?

### 8.7A Problem Solving 3

**Problem 1:** A composite figure is shown below. The figure is made of a cylinder and two cones. Each cone has a height of 10 units and a radius of 6 units. The cylinder has a height of 12 units and a radius of 6 units.



Find the volume of the cylinder center of the figure. Show your work.

Find the volume of each cone that is the end of the figure. Show your work.

Find the volume of the composite figure.

**Problem 2:** A cylinder has a radius of 8 inches and a height of 18 inches. What is the volume of the cylinder?



### TEKS CATEGORY 3 8.8D

### 8.8D Problem Solving 1

**Problem 1:** Look at the two parallel lines, *a* and *b*, and the transversal, *c*, below.



- Give an informal argument for  $\angle 1 \cong \angle 3$ .
- Give an informal argument for if the measure of  $\angle 1$  is  $6x^{\circ}$  and the measure of  $\angle 6$  is  $3x^{\circ}$ , then the measure of  $\angle 1$  is 120°.

**Problem 2:**  $\triangle ABC$  is shown below with exterior angle ACD.



Give an informal argument that the measure of  $\angle ACD$  is equal to the sum of the measures of  $\angle 1$  and  $\angle 2$ .

### 8.8D Problem Solving 2

**Problem 1:** Give an informal argument that if the base angles of an isosceles triangle measure 35° each, then the triangle is an obtuse triangle.

**Problem 2:** Give an informal argument that if two angles of a triangle measure  $(10x + 5)^\circ$ ,  $(5x)^\circ$  and  $(20x)^\circ$ , the smallest angle will NOT measure 35°.

**Problem 3:** In  $\triangle ABC$ , the measure of  $\angle A$  is 25° and the measure of  $\angle B$  is 75°. In  $\triangle DEF$ , the measure of  $\angle E$  is 75° and the measure of  $\angle F$  is 80°. Give an informal argument that the two triangles are similar.



### **TEKS CATEGORY 4** Two Dimensional Shapes



### **TEKS CATEGORY 4** 8.10C

### 8.10C Problem Solving 1

In your own words, describe what each algebraic representation means in terms of a transformation.

- a.  $(x, y) \to (x+2, y-3)$
- b.  $(x, y) \rightarrow (2x, 2y)$
- C.  $(x, y) \rightarrow (y, -x)$
- d.  $(x, y) \rightarrow (-x, -y)$
- e.  $(x, y) \rightarrow (-y, x)$

Is a 270° counterclockwise rotation the same as a 90° clockwise? Explain.

If (3, 2) and (6, 5) are transformed to (2, -3) and (5, -6), what transformation was used?

### 8.10C Problem Solving 2

Triangle ABC has vertices A(1, 5), B(4, 10), and C(6, -1).

Give the coordinates of A' if triangle ABC is transformed as described below.

- a. Translated by  $(x, y) \rightarrow (x+2, y-3) A'($ , )
- b. Reflected over the *y*-axis A'( , )
- c. Reflected over the *x*-axis *A*'( , )
- d. Rotated 90° clockwise about the origin A'(, )
- e. Rotated 180° clockwise about the origin A'(, )
- f. Dilated with a scale factor of 1.5 with the origin as the center of dilation A'(, )

What would be the coordinates of B'' if the triangle is translated 3 units left and then reflected across the *x*-axis?



### **TEKS CATEGORY 5** Measurement and Data



## **TEKS CATEGORY 5** 8.11B

### 8.11B Problem Solving 1

**Problem 1:** Describe how to determine the mean absolute deviation for a set of data points.

**Problem 2:** Give the value of the following quantities:

$$|32-40|$$
  $|52-18|$   $|28-28|$ 

**Problem 3:** Find the mean absolute deviation for the following sets of data: (Show your work.)

- {10, 15, 20, 30, 28, 17}
- {12, 24, 30, 50, 28, 55, 25}
- {2, 20, 34, 5, 25, 55, 26, 17}

### 8.11B Problem Solving 2

**Problem 1:** The chart below shows the heights of the starting 5 players on the grade 8 girls' basketball team at Meadow Middle School.

Heights of the Girls' Basketball					
Team					
5 ft 4 in.	5 ft 4 in. 6 ft				
5 ft 6 in. 5 ft 3 in.					

- What is the mean height of the starting 5 girls on the basketball team?
- What is the mean absolute deviation of the heights?
- What does the mean absolute deviation mean?

**Problem 2:** What does a small mean absolute deviation for a set of data points indicate?

### 8.11B Problem Solving 3

Marcia was asked to create two data sets with the following criteria:

- 1) Set A has a larger mean than Set B.
- 2) Set B has a larger mean absolute deviation than Set A.
- 3) Each set of data has 5 data points.

Marcia created the following sets:

Set A: {10, 20, 30, 40, 50} Set B: {12, 16, 18, 22, 24}

- Do the sets she created meet the criteria she was given? Explain your answer.
- If you answered no, change one of the sets so that the criteria are met, if possible.



### **TEKS CATEGORY 6** Personal Financial Literacy



### TEKS CATEGORY 6 8.12D

### 8.12D Problem Solving 1

**Problem 1:** What is compound interest and how does it differ from simple interest?

What are some common time intervals for compounding interest?

If you use the formula  $A = P(1 + r)^t$  to calculate the amount in an account, what does each variable represent?

**Problem 2:** If a person invested \$1,000 at 4% simple interest and \$1,000 at a rate of 4% compounded annually, what would be the difference in the amounts in each account at the end of the first year? At the end of the second year? (Assume no new deposits.) Show your work.

### 8.12D Problem Solving 2

Complete the table below to show a simple annual interest rate of 6%. Remember the interest is only paid on the original balance unless there was additional money added besides the interest.

Year	Beginning Balance	Interest Rate	Amount of Interest	Ending Balance
1	\$1,000	6%	\$60.00	\$1,060
2	\$1,060			
3				
4				

Complete the table below to show an annual rate of 6% compounded annually.

Year	Beginning	Interest	Amount of	Ending
	Balance	Rate	Interest	Balance
1	\$1,000	6%	\$60.00	\$1,060
2	\$1,060			
3				
4				

How are the charts different in year 2?

What is the difference in the balance after year 4?