TEKSING TOWARD STAAR

MATHEMATICS ®

GRADE 4

Lesson Projections

Six Weeks 1

Lesson 1

Whole Number Place-Value

Every **digit** in a number has a value.

Digits are the symbols used to represent whole numbers. The digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

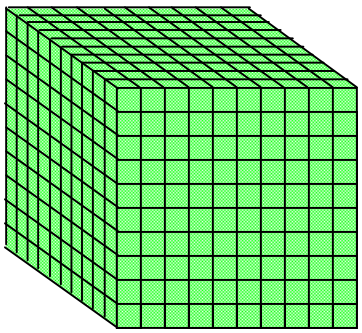
The position, or place, a digit is in tells you the value of the digit. This value is called **place-value**.

Place-Value Patterns

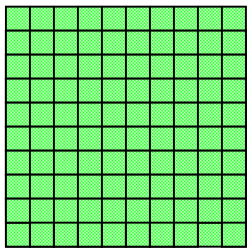
Our number system has patterns that makes it easy to use.

EXAMPLE

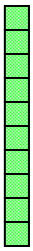
In our number system, each **place** has ten times the **value** of the place to its right.



1 Thousand
10 times greater than 1 hundred



1 Hundred
10 times greater than 1 ten



1 Ten
10 times greater than 1 one

1 One

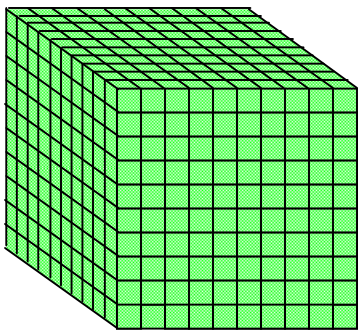
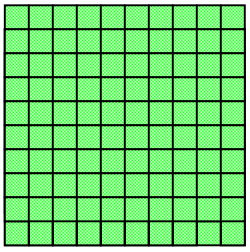
Place-Value Pattern of Tens

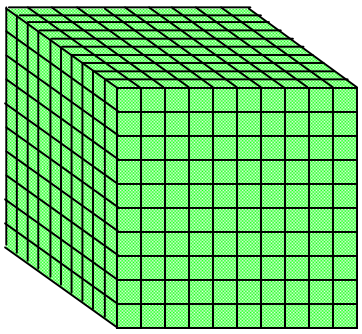
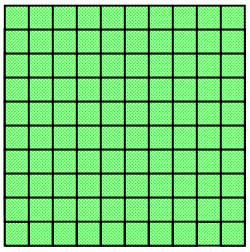
Our number system is based on a simple pattern of tens.

- Each place-value position is ten times the position to its right.
- Each place-value position is one-tenth of the value of the position to its left.

EXAMPLE 1

Base-10 blocks can be used to model the pattern of tens relationships among whole number place-value positions.

Value	1,000	100	10	1
Model				
Description	thousand cube	hundred flat	ten rod	one cube

Value	1,000	100	10	1
Model				
Description	thousand cube	hundred flat	ten rod	one cube

Each place-value position is ten times the value of the position to its right.

- The ten rod is 10 times as much as the unit cube.
- The hundred flat is 10 times as much as the ten rod.
- The thousand cube is 10 times as much as the hundred flat.

Each place-value position is one tenth the value of the position to its left.

- The hundred flat is $\frac{1}{10}$ of the thousand cube.
- The ten rod is $\frac{1}{10}$ of the hundred flat.
- The one cube is $\frac{1}{10}$ of the ten rod.

EXAMPLE 2

Place-value patterns can be used to write numbers that are 10 times as much as or $\frac{1}{10}$ of any given number.

Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
		4,000	400	40	

10 times as much as

$\frac{1}{10}$ of

Each place-value position is ten times the value of the position to its right.

- 4,000 is 10 times as much as 400.

Each place-value position is one tenth the value of the position to its left.

- 400 is $\frac{1}{10}$ of 40.

EXAMPLE 3

A place-value chart can be used to complete a table to record 10 times as much as or $\frac{1}{10}$ of any given number.

Given Number	10 times as much as given number	$\frac{1}{10}$ of given number
6,000		
20		
50,000		
300		

Step 1: Write the given number in a place-value chart.

Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
		6,000			
				20	
	50,000				
			300		

Hundred Thousands	Ten Thousands	One Thousands	Hundreds	Tens	Ones
		6,000			
				20	
	50,000				
			300		

Step 2: Use the place-value chart to write a number in the table that is 10 times as much as the given number.

Given Number	10 times as much as given number	$\frac{1}{10}$ of given number
6,000	60,000	
20	200	
50,000	500,000	
300	3,000	

Step 3: Use the place-value chart to write a number in the table that is $\frac{1}{10}$ of the given number.

Given Number	10 times as much as given number	$\frac{1}{10}$ of given number
6,000	60,000	600
20	200	2
50,000	500,000	5,000
300	3,000	30

Place-Value Patterns to Read and Write Whole Numbers

Our number system arranges numbers into groups of three places called **periods**.

The places within the periods repeat (hundreds, tens, ones, hundreds, tens ones, and so on.)

BILLIONS PERIOD			MILLIONS PERIOD			THOUSANDS PERIOD			ONES PERIOD		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
		1	0	0	0	0	0	0	0	0	0

In the United States, we usually use commas to separate the periods.

The number represented in the place-value chart is 1,000,000,000.

This number is read as "one billion."

Fourth grade students are expected to represent the value of whole numbers through 1,000,000,000.

Knowing the place and period of a number will help you find the value of digits in any number, as well as read and write numbers.

EXAMPLE

987,654,321

BILLIONS PERIOD			MILLIONS PERIOD			THOUSANDS PERIOD			ONES PERIOD		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
			9	8	7	6	5	4	3	2	1

The digit **9** is in the *hundred millions* place.
The value of the digit is 900,000,000.

The digit **8** is in the *ten millions* place.
The value of the digit is 80,000,000.

The digit **7** is in the *one millions* place.
The value of the digit is 7,000,000.

The digit **6** is in the *hundred thousands* place.
The value of the digit is 600,000.

The digit **5** is in the *ten thousands* place.
The value of the digit is 50,000.

The digit **4** is in the *one thousands* place.
The value of the digit is 4,000.

The digit **3** is in the *hundreds* place.
The value of the digit is 300.

The digit **2** is in the *tens* place.
The value of the digit is 20.

The digit **1** is in the *ones* place.
The value of the digit is 1.

Standard Form and Word Form of Whole Numbers

- A number written with one digit for each place-value is written in **standard form**.

  The standard form for the number *three thousand three* is 3,003.

- A number written with words is written in **word form**.

  The word form for 3,003 is *three thousand three*.

Fourth grade students should be able to read and write numbers in **word form**, **standard form**, and **expanded form**.

You will learn about expanded form later in this lesson.

Place-Value to Read and Write Whole Numbers

When you read numbers, always start on the left.

Many numbers have more than three digits. The digits in these numbers are arranged in groups of three called **periods**. A comma is used to separate each **period**.

MILLIONS PERIOD			THOUSANDS PERIOD			ONES PERIOD		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
4	8	5	1	0	2	2	9	9

485,102,299 is shown in the place-value chart.

A comma separates the millions period from the thousands period. 485,102,299

A comma separates the thousands period from the ones period. 485,102,299

EXAMPLE 1

Read and write 45,073 in **word form**.

Look at 45,073 in the place-value chart.

MILLIONS PERIOD			THOUSANDS PERIOD			ONES PERIOD		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
			3	0	4	9	2	7

This is a five-digit number.

To read this number:

- first, say the two-digit number to the left of the comma, ***forty-five***;
- next, say the name of the period, ***thousand*;**
- then, say the three-digit number to the right of the comma, ***seventy-three***.

45,073 is read as

forty-five thousand, seventy-three.

This is the **word form** of the number.

Problem-Solving Model

Step	Description of Step
1	**Analyze the given information.** •Summarize the problem in your own words. •Describe the main idea of the problem. •Identify information needed to solve the problem.
2	**Formulate a plan or strategy.** •Draw a picture or a diagram. •Find a pattern. •Guess and check. •Act it out. •Create or use a chart or a table. •Work a simpler problem. •Work backwards. •Make an organized list. •Use logical reasoning. •Brainstorm. •Write a number sentence or an equation.
3	**Determine a solution.** •Estimate the solution to the problem. •Solve the problem.
4	**Justify the solution.** •Explain why your solution solves the problem.
5	**Evaluate the process and the reasonableness of your solution.** •Make sure the solution matches the problem. •Solve the problem in a different way.

Problem-Solving Questions

Directions:

- **Work with a partner.**
- **Write your answers on notebook paper.**
- **Answer questions 1-3.**
- **Complete the solution to the problem.**
- **Answer questions 4-10.**

1. What is the main idea of this problem?
2. What are the supporting details in this problem?
3. What skills, concepts and understanding of math vocabulary are needed to be able to answer this problem?
4. Did this problem involve mathematics arising in everyday life, society, or the work place?
5. What is a good problem solving strategy for this problem?
6. Can you explain how you used any math tools, mental math, estimation or number sense to solve this problem?
7. Did this problem involve using multiple representations (symbols, diagrams, graphs, math language)?
8. Did you use any relationships to solve this problem?
9. How can you justify your solution to the problem?
10. How can you check for reasonableness of your solution to this problem?

Problem-Solving 1

Work with a partner. Your teacher will give you and your partner 9 number cards.

1. Create the greatest number possible using all the number cards and placing the 6 card in the ten millions place.
2. Write this number in standard and word form.
3. Read the number out loud to your partner in standard form and in word form.
4. Create the least number possible using all the number cards and placing the 6 card in the ten thousands place.
5. Write this number in standard and word form.
6. Read the standard form and word form out loud to your partner.
7. Create two other 9-digit numbers using all the number cards and placing the 6 card in the tens place or the hundreds place.
8. Write the numbers in standard form and in word form.
9. Read the standard form and word form out loud to your partner.

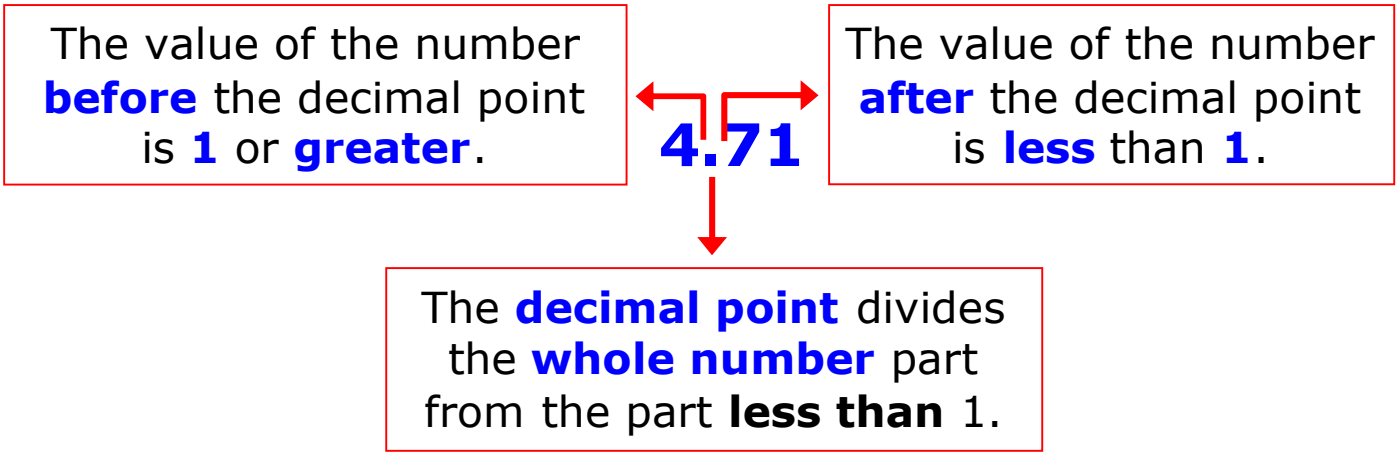
Decimal Place-Value

Every **digit** in a number has a value.

After the ones period in place-value there is a "dot" called a **decimal point**.

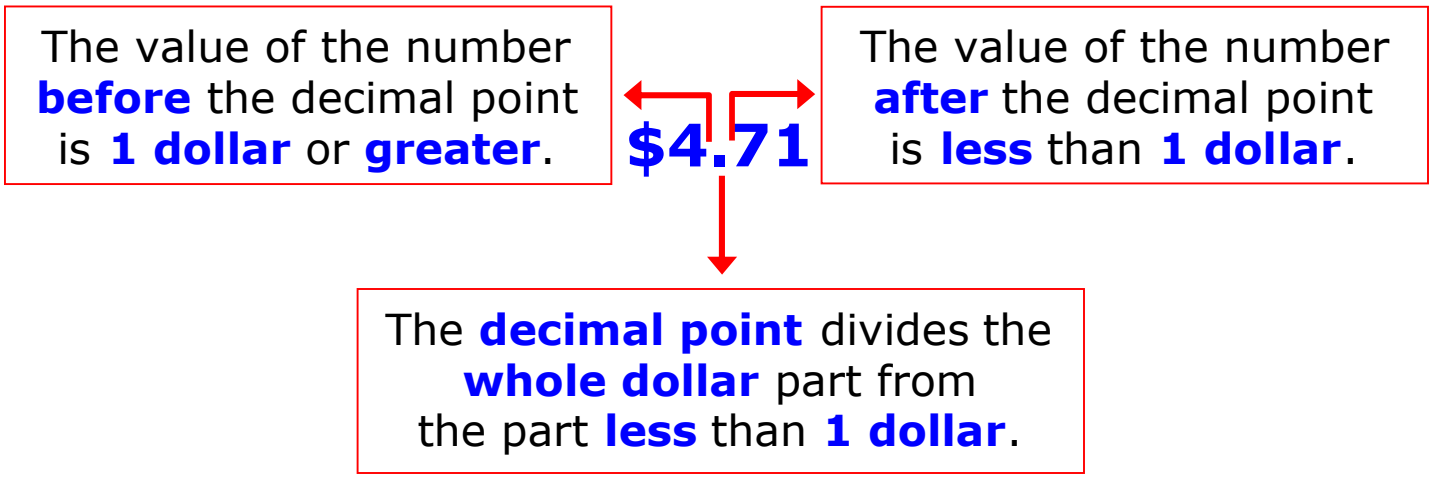
The decimal point is used to separate the whole number part from the part less than one.

The numbers to the right of the **decimal point** are called **decimals**.



Dollar symbols and **decimal points** are used to write money amounts.

One dollar represents 1 whole.

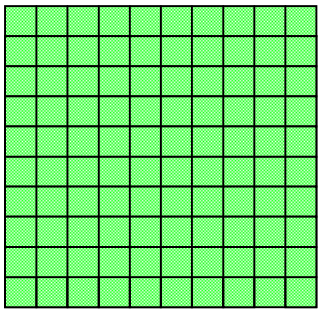


Decimal Place-Value Patterns

Decimals follow the same place-value pattern as whole numbers.

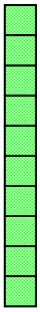
EXAMPLE

Each **place** continues to have the **value** of the place to its right.



1 One

10 times greater than 1 tenth



1 Tenth

10 times greater than 1 hundredth

1 Hundredth

Decimal Place-Value Pattern of Tens

Decimal values are based on the same simple pattern of tens as whole numbers.

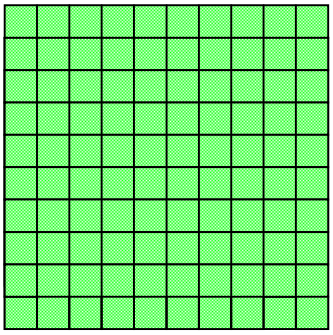
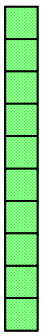
- Each place-value position is ten times the position to its right.
- Each place-value position is one-tenth of the value of the position to its left.

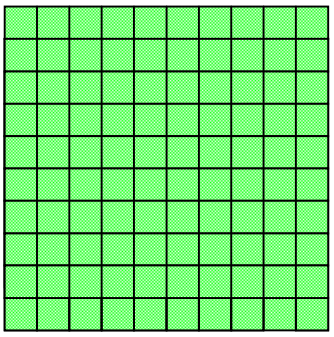
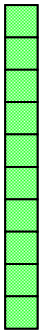
EXAMPLE 1

Base-10 blocks can be used to model the pattern of tens relationships among decimal place-value positions.

The base-10 blocks used to represent decimal place-value positions are some of the same blocks used to represent whole numbers, but their value and description is different for decimals.

- The flat now represents 1 whole, the rod now represents $\frac{1}{10}$, and the small cube now represents $\frac{1}{100}$.

Value	1	$\frac{1}{10}$	$\frac{1}{100}$
Model			
Description	one flat	tenth rod	hundredth cube

Value	1	$\frac{1}{10}$	$\frac{1}{100}$
Model			
Description	one flat	tenth rod	hundredth cube

Each place-value position is ten times the value of the position to its right.

- The one flat is 10 times as much as the tenth rod.
- The tenth rod is 10 times as much as the hundredth cube.

Each place-value position is one tenth of the value of the position to its left.

- The tenth rod is $\frac{1}{10}$ of the one flat.
- The hundredth cube is $\frac{1}{10}$ of the tenth rod.

EXAMPLE 2

Money can be used to model the pattern of tens relationships among decimal place-value positions. A dollar bill represents 1 whole, a dime represents $\frac{1}{10}$, a penny represents $\frac{1}{100}$.

$1.11 is represented in the table.

Value	$1.00	$0.10	$0.01
Model			
Description	dollar	dime	penny

Each place-value position is ten times the value of the position to its right.

- The dollar is 10 times as much as the value of the dime.
- The dime is 10 times as much as the value of the penny.

Each place-value position is one tenth of the value of the position to its left.

- The dime is $\frac{1}{10}$ of the value of the dollar.
- The penny is $\frac{1}{10}$ of the value of the dime.

EXAMPLE 3

Decimal place-value patterns can be used to write numbers that are 10 times as much as or $\frac{1}{10}$ of any given number.

Tens	Ones	.	Tenths	Hundredths
	4	.	4	4

10 times as much as

$\frac{1}{10}$ of

Each place-value position is ten times the value of the position to its right.

- 4 is 10 times as much as 0.4
- 0.4 is 10 times as much as 0.04

Each place-value position is one tenth value of the position to its left.

- 0.04 is $\frac{1}{10}$ of 0.4
- 0.4 is $\frac{1}{10}$ of 4

EXAMPLE 4

A place-value chart can be used to complete a table to record 10 times as much as or $\frac{1}{10}$ of any given number.

Given Number	10 times as much as given number	$\frac{1}{10}$ of given number
6.0		
0.7		
8.0		
0.9		

Step 1

Write the given numbers in a place value chart.

Tens	Ones	.	Tenths	Hundredths
	6	.	0	
	0	.	7	
	8	.	0	
	0	.	9	

Step 2

Tens	Ones	.	Tenths	Hundredths
	6	.	0	
	0	.	7	
	8	.	0	
	0	.	9	

Use the place-value chart to write a number in the table that is 10 times as much as the given number.

Given Number	10 times as much as given number	$\frac{1}{10}$ of given number
6.0	60	
0.7	7.0	
8.0	80	
0.9	9.0	

Step 3

Tens	Ones	.	Tenths	Hundredths
	6	.	0	
	0	.	7	
	8	.	0	
	0	.	9	

Use the place-value chart to write a number in the table that is $\frac{1}{10}$ of the given number.

Given Number	10 times as much as given number	$\frac{1}{10}$ of given number
6.0	60	0.6
0.7	7.0	0.07
8.0	80	0.8
0.9	9.0	0.09

Place-Value Patterns to Read and Write Decimal Numbers

The decimal place-values begin after the ones place and a decimal point.

The first position after the decimal point is the tenths place followed by the next position, the hundredths place.

Tens	Ones	.	Tenths	Hundredths
	1	.	1	1

A decimal point is used to separate the numbers 1 or greater from the numbers less than one.

The number represented in the place-value chart is 1.11.

When you read a decimal number, the decimal point is said as "***and***". This number is read as "*one **and** eleven hundredths*."

Fourth grade students are expected to represent the value of decimals through hundredths. Knowing the place will help you find the value of digits in any number, as well as read and write decimal numbers.

EXAMPLE 1

Write 0.6 on a place-value chart.

(Since this number is less than one, there is a zero in the ones place.)

Tens	Ones	.	Tenths	Hundredths
	0	.	6	

The digit **0** is in the *ones* place. The value of the digit is 0.0.

The digit **6** is in the *tenths* place. The value of the digit is 0.6 (less than 1).

EXAMPLE 2

Write 0.58 on a place-value chart.

Tens	Ones	.	Tenths	Hundredths
	0	.	5	8

The digit **0** is in the *ones* place.
The value of the digit is 0.0.

The digit **5** is in the *tenths* place.
The value of the digit is 0.5 (less than 1).

The digit **8** is in the *hundredths* place.
The value of the digit is 0.08 (less than 1).

EXAMPLE 3

Write 9.47 on a place-value chart.

Tens	Ones	.	Tenths	Hundredths
	9	.	4	7

The digit **9** is in the *ones* place.
The value of the digit is 9.0.

The digit **4** is in the *tenths* place.
The value of the digit is 0.4 (less than 1).

The digit **7** is in the *hundredths* place.
The value of the digit is 0.07 (less than 1).

EXAMPLE 4

Write 29.3 on a place-value chart.

Tens	Ones	.	Tenths	Hundredths
2	9	.	3	

The digit **2** is in the *tens* place.
The value of the digit is 20.0.

The digit **9** is in the *ones* place.
The value of the digit is 9.0.

The digit **3** is in the *tenths* place.
The value of the digit is 0.3 (less than 1).

Standard Form and Word Form of Decimal Numbers

- A number written with one digit for each place-value is in **standard form**.

  The standard form for the number *six tenths* is 0.6.

  The standard form for the number *fifty-eight hundredths* is 0.58.

  The standard form for the number *nine and forty-seven hundredths* is 9.47.

  The standard form for the number *twenty-seven and three hundredths* is 27.03.

- A number written in words is in **word form**.

  The word form for 0.6 is *six tenths*.

  The word form for 0.58 is *fifty-eight hundredths*.

  The word form for 9.47 is *nine and forty-seven hundredths*.

  The word form for 27.03 is *twenty-seven and three hundredths*.

Fourth grade students should be able to read and write decimal numbers in word form, standard form, and expanded form. You will learn about expanded form later in this lesson.

Place-Value to Read and Write Decimal Numbers

When you read numbers, always start on the left.

The number 38.65 is shown in the place-value chart.

Tens	Ones	.	Tenths	Hundredths
3	8	.	6	5

A decimal point separates the part of the number that is greater than 1 from the part of the number that is less than 1.

38.65 is the **standard form**.

The word form of 38.65 is written and said as *thirty-eight and sixty-five hundredths*.

(When you write or say a decimal number, the decimal point is written and said as "**and**".)

EXAMPLE 1

Read and write 0.73 in **word form**.

Look at 0.73 in the place-value chart.

Tens	Ones	.	Tenths	Hundredths
	0	.	7	3

To read this number:

- first, say the two-digit number to the right of the decimal point ***seventy-three***;
- then, say the name of the last place to the right, ***hundredths***.

0.73 is read as ***seventy-three hundredths***.

This is the **word form** of the number.

EXAMPLE 2

Read and write 6.07 in **word form**.

Look at 6.07 in the place-value chart.

Tens	Ones	.	Tenths	Hundredths
	6	.	0	7

To read this number:

- first, say the one-digit number to the left of the decimal point ***six***;
- then, say ***and***;
- next, say the two-digit number to the right of the decimal point ***seven***;
- then, say the name of the last place to the right, ***hundredths***.

6.07 is read as ***six and seven hundredths***.

This is the **word form** of the number.

EXAMPLE 3

Read and write 13.48 in **word form**.

Look at 13.48 in the place-value chart.

Tens	Ones	.	Tenths	Hundredths
1	3	.	4	8

To read this number:

- first, say the two-digit number to the left of the decimal point ***thirteen***;
- then, say ***and***;
- next, say the two-digit number to the right of the decimal point ***forty-eight***;
- then, say the name of the last place to the right, ***hundredths***.

13.48 is read as ***thirteen and forty-eight hundredths***.

This is the **word form** of the number.

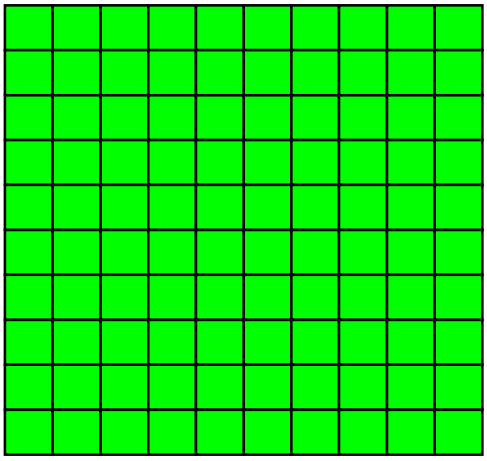
Problem-Solving 2

Work with a partner. Your teacher will give you and your partner 8 number cards and a decimal point card.

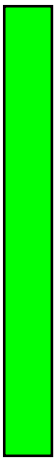
1. Create the largest number possible using four number cards, placing the decimal point card after a 2-digit whole number, and placing the 3 card in the tens place.
2. Write and read this number in standard and word form.
3. Create the smallest number possible using four number cards, placing the decimal point after a two-digit whole number, and placing the 3 card in the hundredths place.
4. Write and read this number in standard and word form.
5. Write two numbers using 5 of the number cards, the decimal point after a three-digit whole number, and the 0 card in the tenths or hundredths place.
6. Write and read the numbers in standard form and in word form.

Decimal Place-Value Models

- Base-ten blocks can be used to model decimal place-value.
- The flat represents 1.



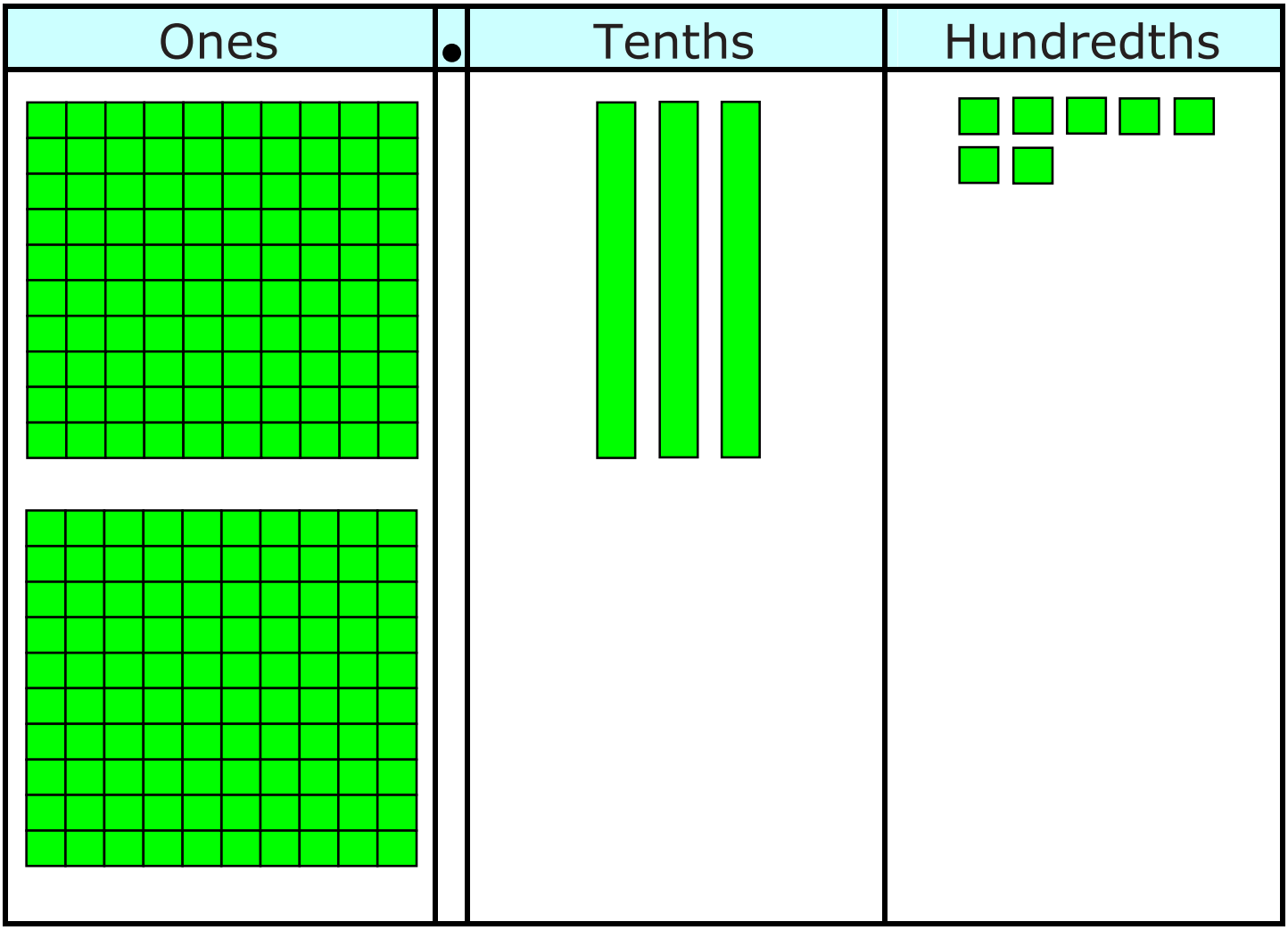
- The rod represents 1 tenth or 0.1.



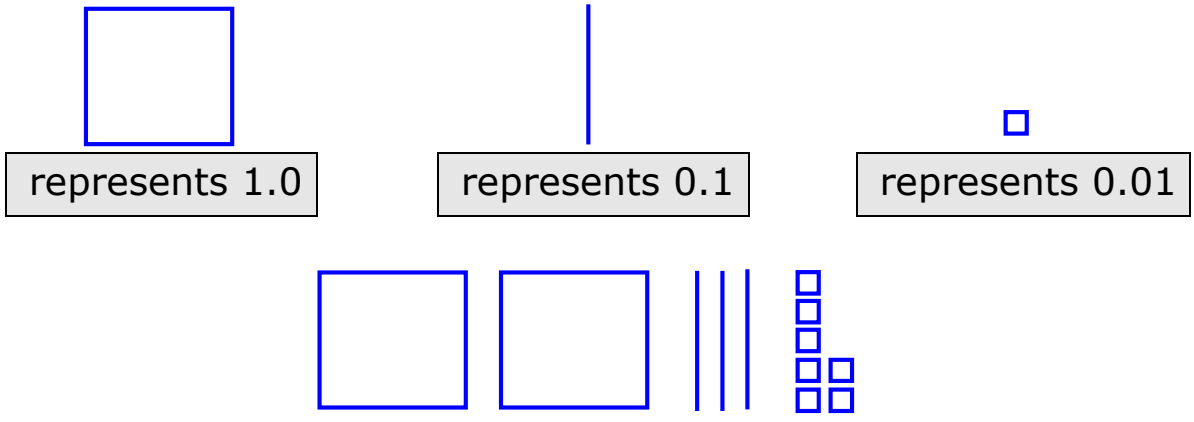
- The small cube represents 1 hundredth or 0.01.

Decimal Place-Value Model Mat

Use base-10 blocks to model 2.37.



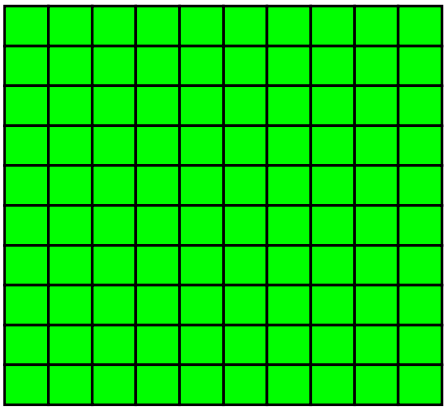
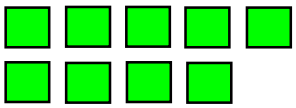
Use symbols to make a quick sketch of the model.



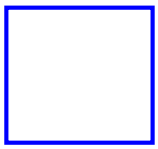
The model shows 2.37 in standard form and *two and thirty-seven hundredths* in word form.

Decimal Place-Value Model Mat

Use base-10 blocks to model 1.09

Ones	.	Tenths	Hundredths
			

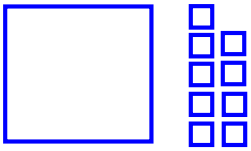
Use symbols to make a quick sketch of the model.



represents 1.0

represents 0.1

represents 0.01



The model shows 1.09 in standard form and *one and nine hundredths* in word form.

Writing Numbers in Expanded Notation

Writing numbers in **expanded form**, or **expanded notation**, is another way to help you understand place-value.

Expanded form or expanded notation is a way to write numbers to show the value of each digit.

EXAMPLE 1

Write 904,586 in expanded notation.

Look at 904,586 in the place-value chart.

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
9	0	4	5	8	6

$9 \times 100{,}000 + 0 \times 10{,}000 + 4 \times 1{,}000 + 5 \times 400 + 8 \times 10 + 6 \times 1$

The chart shows the value of each digit.

- The digit 9 is in the hundred thousands place so it represents 9 hundred thousand and has a value of 900,000.
- The digit 0 is in the ten thousands place so it represents 0 ten thousands and has a value of 0.

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
9	0	4	5	8	6

$9 \times 100{,}000 + 0 \times 10{,}000 + 4 \times 1{,}000 + 5 \times 400 + 8 \times 10 + 6 \times 1$

- The digit 4 is in the thousands place so it represents 4 thousands and has a value of 4,000.
- The digit 5 is in the hundreds place so it represents 5 hundreds and has a value of 500.
- The digit 8 is in the tens place so it represents 8 tens and has a value of 80.
- The digit 6 is in the ones place so it represents 6 ones and has a value of 6.

The value of the number 904,586 is

900,000 + 0 + 4,000 + 500 + 80 + 6

EXAMPLE 2

Write the number 94.56 in expanded notation.

Look at 94.56 in the place-value chart.

Tens	Ones	.	Tenths	Hundredths
9	4	.	5	6

$9 \times 10 + 4 \times 1 + 5 \times 0.1 + 6 \times 0.01$

The place-value chart shows the value of each digit.

- The digit 9 is in the tens place so it represents 9 tens and has a value of 90.
- The digit 4 is in the ones place so it represents 4 ones and has a value of 4.
- The digit 5 is in the tenths place so it represents 5 tenths and has a value of 0.05
- The digit 6 is in the hundredths place so it represents 6 hundredths and has a value of 0.06

The value of the number 94.56 is

$$90 + 4 + 0.5 + 0.06$$

Problem-Solving 3

The table below shows the lengths of several different types of sharks a marine biology class measured off the shore in Galveston, Texas.

Type of Shark	Length
Bonnethead	0.9 meters
Blackfin	1.78 meters
Sandbar	1.5 meters
Blacknose	0.83 meters

1. What is the expanded notation for the length of the bonnethead shark?

2. What is the expanded notation for the length of the blackfin shark?

3. What is the expanded notation for the length of the sandbar shark?

4. What is the expanded notation for the length of the blacknose shark?

5. What is the word form for the length of the bonnethead shark?

6. What is the word form for the length of the blackfin shark?

Type of Shark	Length
Bonnethead	0.9 meters
Blackfin	1.78 meters
Sandbar	1.5 meters
Blacknose	0.83 meters

7. What is the word form for the length of the sandbar shark?

8. What is the word form for the length of the blacknose shark?

9. What is the value of the digit 8 in the length of the blackfin shark?

10. What is the value of the digit 3 in the length of the blacknose shark?

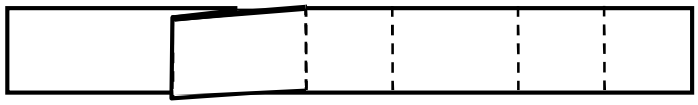
11. What is the value of the digit 1 in the length of the sandbar shark?

12. What is the value of the digit 9 in the length of the bonnethead shark?

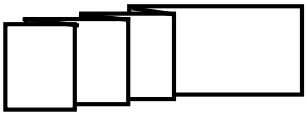
Decimal Expanded Form Strips Instructions

- Cut out each of the 4 strips on the page. Cut along the dark lines only.
- Place 1 strip in front of you on your work area. Place the largest rectangle to the left.

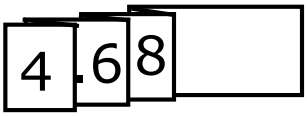
- Fold along the first two dashed lines on the left. Keep the largest rectangle to the left.



- Continue folding along the dashed lines, then turn the strip around so that the largest rectangle on the right.



- Write 4, 6 and 5 on the strip, writing one digit in each place as shown. Place a decimal point before the 6.



- Unfold your strip. Use numbers and symbols to write 4 + 0.6 + 0.08 as shown. Your strip represents the expanded form of 4.68

4	+	0.6	+	0.08	=	4.68

- Use your last 3 strips to represent the expanded form of: 9.57 5.53 2.09

Lesson 6

Describing Relationships

Relationships can be described mathematically by replacing words and sentences with numbers, symbols, expressions, and equations. Describing relationships with numbers, symbols, expressions, and equations can help to solve problems.

Symbols, Variables, and Expressions

- A **symbol** is something that represents something else in mathematics.

  The symbol **+** means add.

  The symbol **–** means subtract

- If something varies, that means it changes. Most things, like your height and weight, do not stay the same.

  In mathematics, to describe things that change, or vary, letters are used instead of numbers. When a letter is used this way, it is called a **variable**. Any letter in the alphabet can be used as a **variable**.

EXAMPLES

n (number of inches tall you are)
t (amount of time you spend on homework)
c (number of cents in your pocket)

- In language, an expression can be a short way to describe an idea or feeling.

  In mathematics, an **expression** is a short way to describe an amount.

  An **expression** is a **variable** or combination of variables, numbers, and symbols that represents a mathematical relationship.

EXAMPLES

Sometimes an expression is just a number, like 6.

Sometimes an expression is just a variable, like w.

Sometimes an expression is a combination of numbers, variables, and operations, like 6 + 3 or $n - 3$.

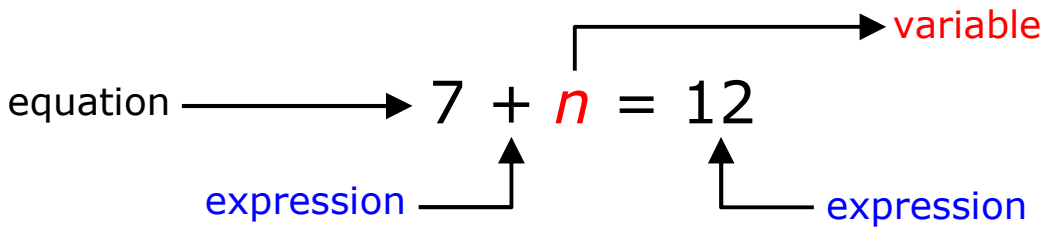
Writing Expressions

To write an expression that describes what is going on in a word problem, think about the problem in words. Use numbers when you know what they are. Use variables when you do not know the numbers.

Problem	Expression in Words	Expression
A fourth grade class has 3 more boys than girls. Write an **expression** to represent how many boys are in the class.	The number of boys is three more than the number of girls. g = number of girls	$g + 3$
A school bus carrying 9 students is at a bus stop. No more students got on, but some of the students got off. Write an **expression** to represent the number of students left on the bus.	The number of students left on the bus is 9 minus the number of students who got off. n = number of students who got off	$9 - n$

Writing Equations

An **equation** is a mathematical sentence that tells you that two expressions are equal.



To write an equation, think about which two amounts are equal in a problem. Then write an expression for each amount.

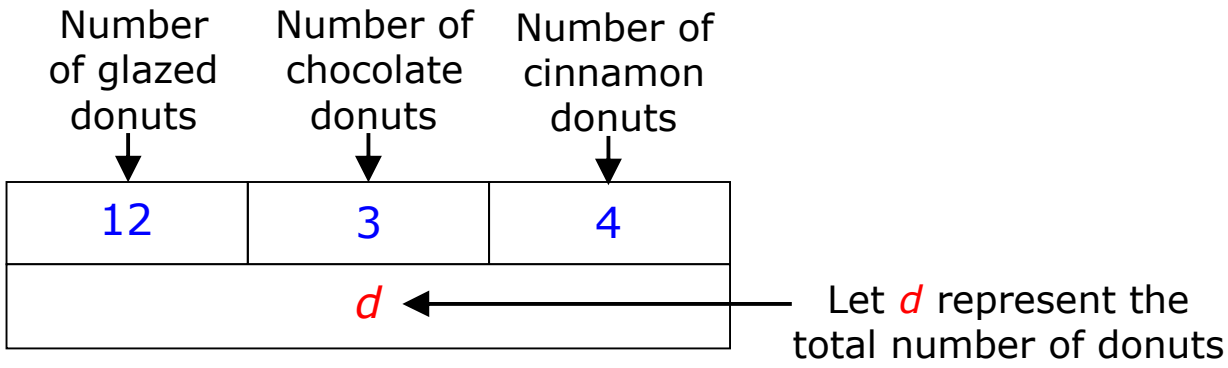
Problem	Equal Expressions in Words	Equation
A gift box weighs 6 ounces when empty. When the gift is placed in the inside, the box weighs 28 ounces. Write an equation to represent the weight of the gift.	6 ounces + weight of gift = 28 ounces	$6 + w = 28$
Elliott gave away 7 of her stuffed animals. She has 35 animals left. Write an equation to represent the number of stuffed animals Elliott has before she gave 7 away.	total animals – 7 animals = 35 animals	$a - 7 = 35$

Strip Diagrams

A strip diagram can be used to represent addition or subtraction problem situations.

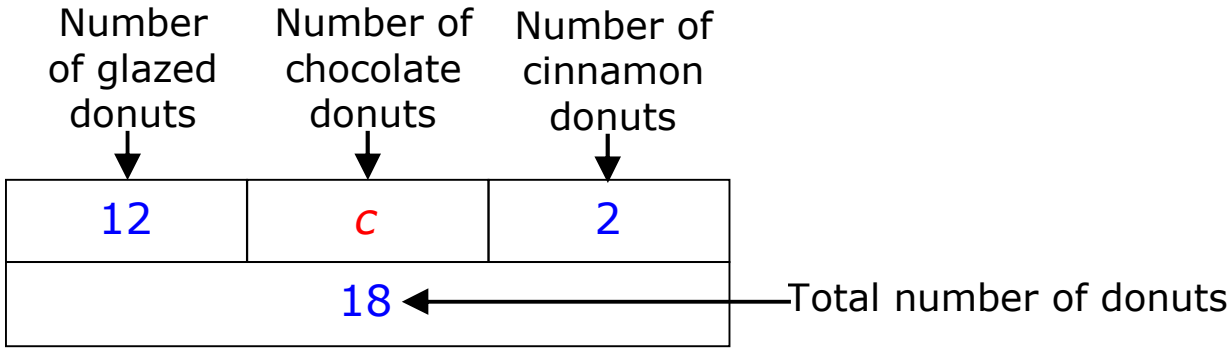
EXAMPLE 1

Michelle has one dozen glazed donuts, 3 chocolate covered donuts, and 4 cinnamon covered donuts. How many donuts does Michelle have?



EXAMPLE 2

Miguel has 18 donuts. He has one dozen glazed donuts, some chocolate donuts, and 2 cinnamon donuts. How many chocolate donuts does he have?

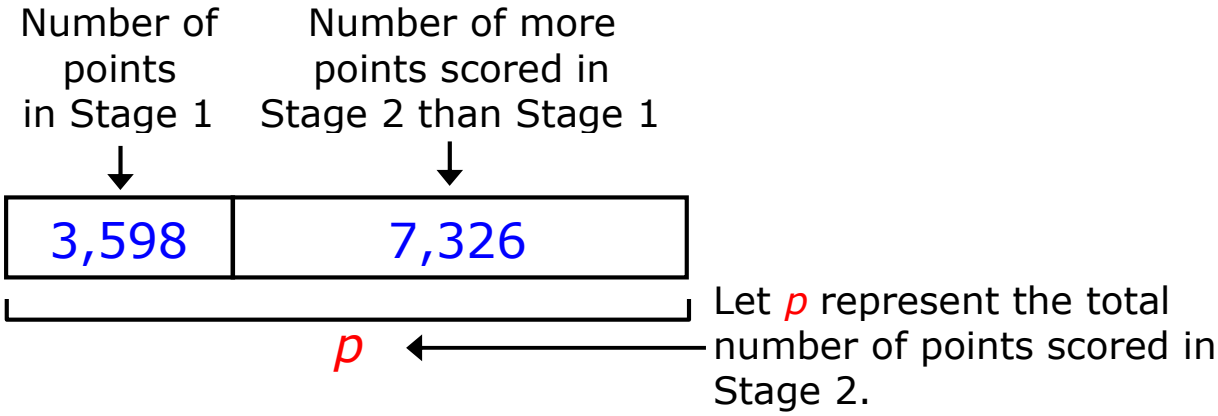


EXAMPLE 3

Jackson scored 3,598 points in Stage 1 of a new computer game. He scored 7,326 more points in Stage 2 than he scored in the Stage 1. How many total points did Jackson score in the game?

- Find how many points Jackson scored in Stage 2.

  Use a strip diagram to represent the number of points Jackson scored in Stage 2.



Write an equation.

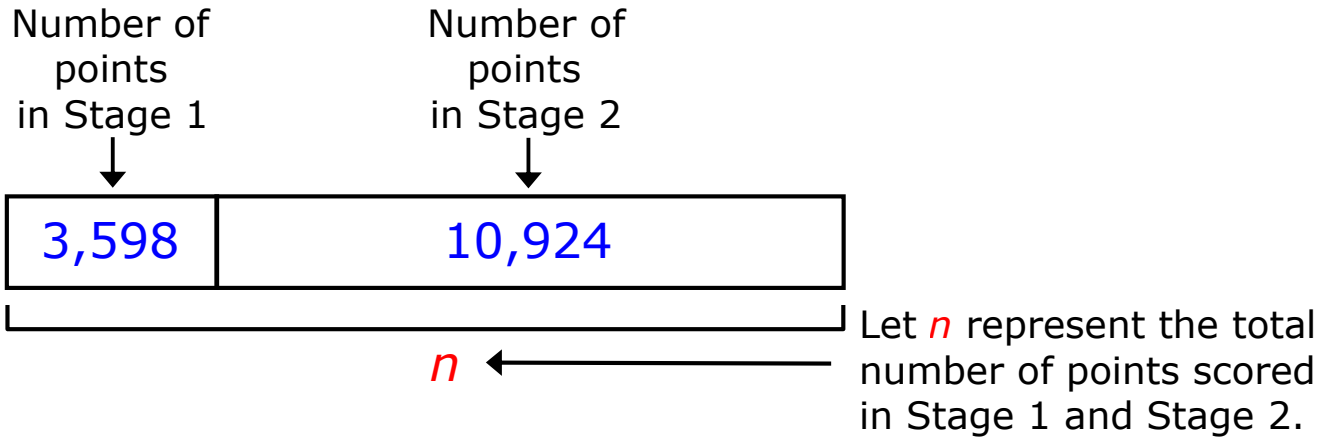
$$3{,}598 + 7{,}326 = p$$

Solve the equation.

$$10{,}924 = p$$

- Find how many total points Jackson scored in the game.

  Use a strip diagram to represent the total points Jackson scored in the game.



Write an equation.

$$3{,}598 + 10{,}924 = n$$

Solve the equation.

$$14{,}522 = n$$

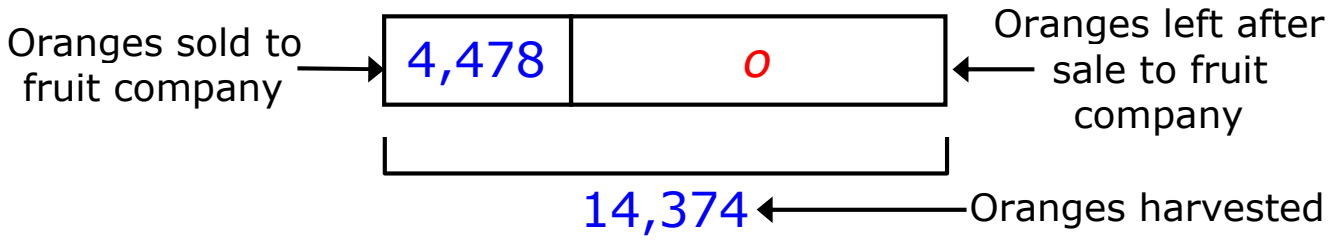
So, Jackson scored 14,522 points in the game.

EXAMPLE 4

During the citrus harvest in the Texas Valley, 14,374 oranges were picked on a citrus farm. First, he sold 4,478 oranges to a fruit company. Then he sold 3,224 oranges in his fruit stands. How many oranges are left to sell from this harvest?

- Find how many oranges were left to sell.

Use a strip diagram to represent the number of oranges left after the sale to the fruit company.



Write an equation.

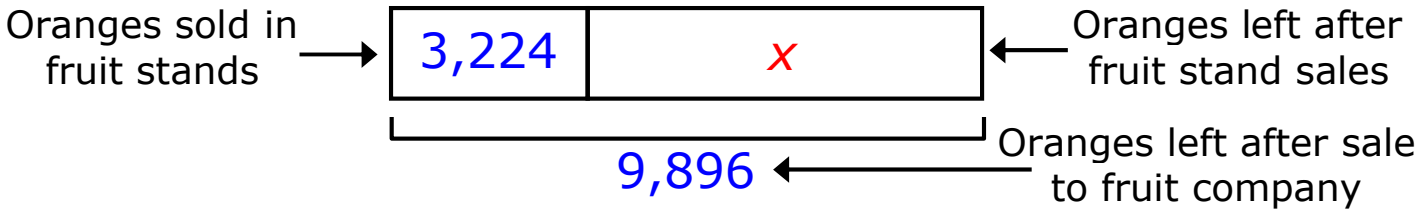
$$14{,}347 - 4{,}478 = o$$

Solve the equation.

$$9{,}896 = o$$

- Find how many oranges were left after the sale to the fruit company.

  Use a strip diagram to represent the number of oranges left after the fruit stand sales.



Write an equation.

$$9{,}896 - 3{,}224 = x$$

Solve the equation.

$$6{,}672 = x$$

So, 6,672 oranges are left to sell from this harvest.

Problem-Solving 1

PROBLEM 1

Bertie had 48 yards of fabric. She used 12 yards when she made a quilt. Then she used 28 yards when she made some curtains.

1. Make a strip diagram to represent how much fabric Bertie has left.

2. Write an equation to represent how much fabric Bertie has left.

PROBLEM 2

Carlos has 19 baseball cards, and Dan has 11 more baseball cards than Carlos. Tommy has 8 less baseball cards than Dan has.

1. Make a strip diagram to represent the number of baseball cards that Tommy has.

2. Write an equation to represent the number of baseball cards that Tommy has.

Number Patterns

A **number pattern** is set of numbers that is related to each other by a specific rule.

Each number in the pattern is called a **term**.

Each term in the pattern has a **position** and each term has a **value**.

The pattern can be described by a specific **rule**.

A number pattern can be represented by a **sequence** of numbers.

EXAMPLE 1

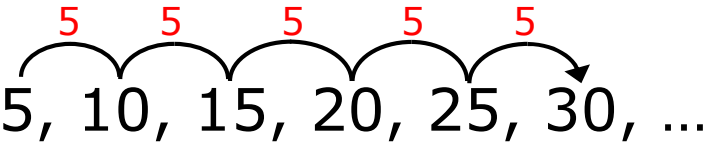
5, 10, 15, 20, 25, 30, … is a number pattern. Find and describe the rule for this number pattern.

- Decide if each number in the pattern is greater or less than the number before it.

5, 10, 15, 20, 25, 30, …

Each number in this pattern is greater than the number before it.

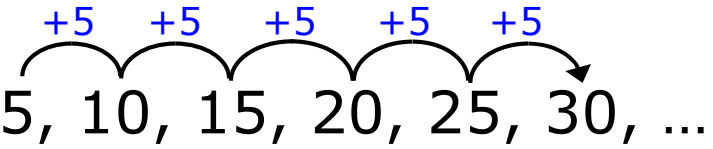
- Record the difference between the numbers in the pattern.

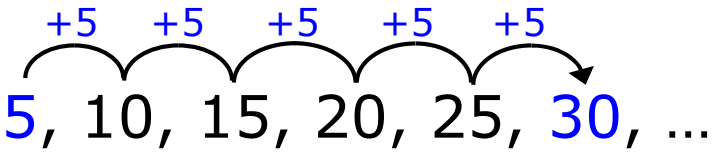


The difference between the numbers in this pattern is 5.

- Decide the rule for the pattern.

The rule for this pattern is *add* 5.





The first term in this pattern is 5.

The sixth term in this pattern is 30.

To find the next term, or the seventh term in this pattern, *add* 5.

The next term in this pattern is 30 + 5, so the next term is 35.

EXAMPLE 2

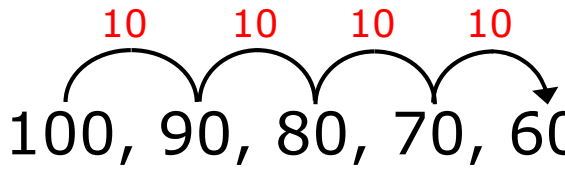
100, 90, 80, 70, 60, … is a number pattern. Find and describe the rule for this number pattern.

- Decide if each number in the pattern is greater or less than the number before it.

  100, 90, 80, 70, 60, …

  Each number in this pattern is less than the number before it.

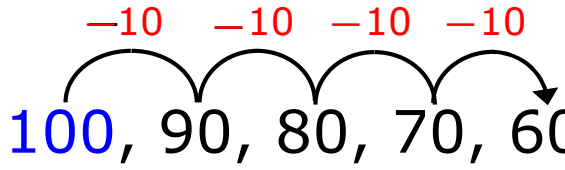
- Decide the difference between the numbers in the pattern.



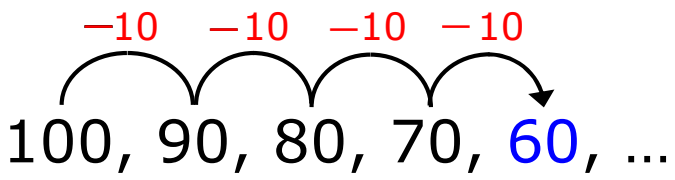
  The difference between the numbers in this pattern is 10.

- Decide the rule for the pattern.

  The rule for this pattern is *subtract* 10.



  The first term in this pattern is 100.



The fifth term in this pattern is 60.

To find the next term, or the sixth term in this pattern, *subtract* 10.

The next term in this pattern is 60 − 10, so the next term is 50.

EXAMPLE 3

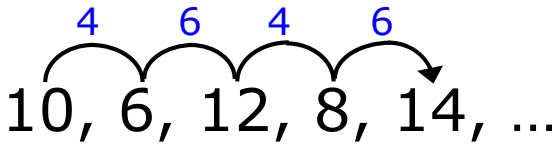
10, 6, 12, 8, 14, … is a number pattern. Find and describe the rule for this number pattern.

- Decide if each number in the pattern is greater or less than the number before it.

10, 6, 12, 8, 14, …

The second number in this pattern is less than the number before it. The third number in this pattern is greater than the number before it.

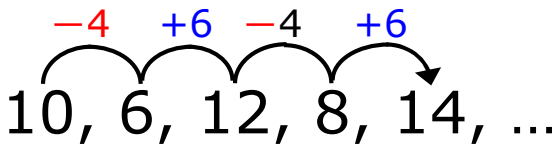
- Decide the difference between the numbers in the pattern.

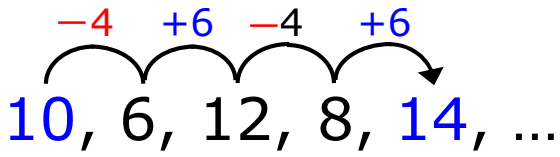


The difference between the numbers in this pattern is 4, 6, 4, 6.

- Decide the rule for the pattern.

The rule for this pattern is *subtract* 4, *add* 6.





The first term in this pattern is 10.

The fifth term in this pattern is 14.

To find the next term, or the sixth term in this pattern, *subtract* 4.

The sixth term in this pattern is 14 − 4, so the sixth term is 10.

To find the next term, or the seventh term in this pattern, *add* 6.

The seventh term in this pattern is 10 + 6, so the seventh term is 16.

Generating a Number Pattern

An **input-output table** can be used to generate a pattern.

A pattern is called a **function** when one quantity depends on the other.

An input-output table shows the **relationship** between the inputs and outputs of a function.

A **rule** can be written to describe this relationship.

The rule can be an expression or an equation.

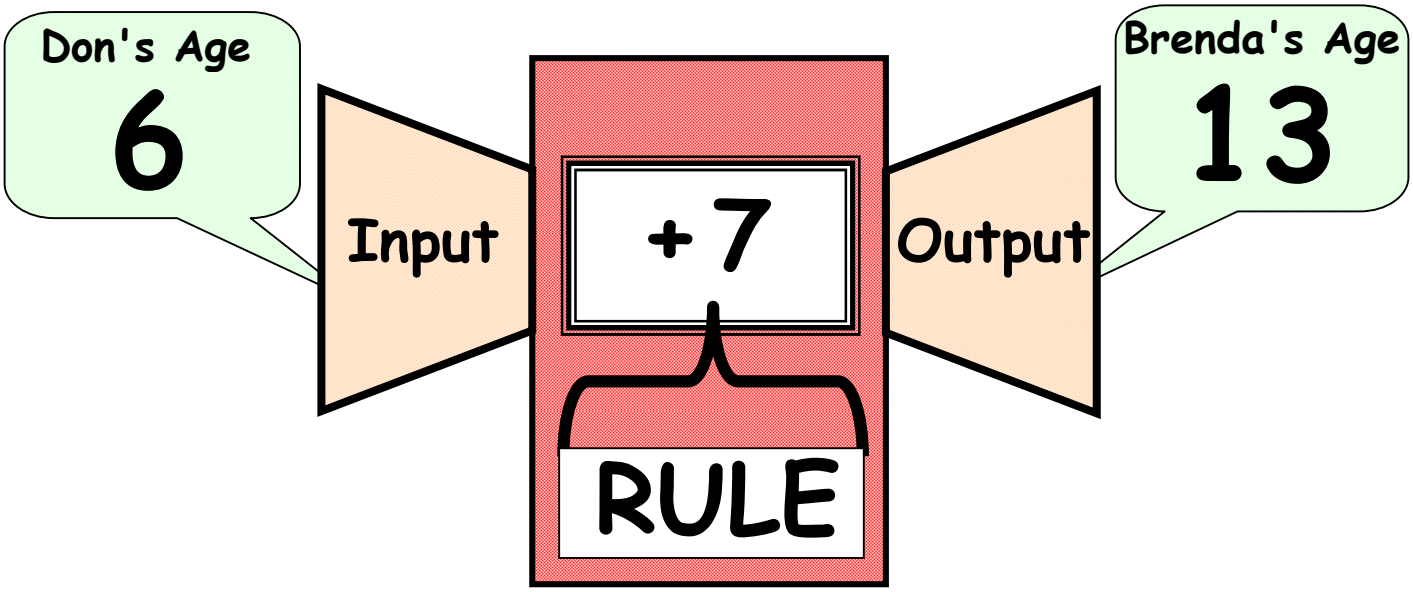
EXAMPLE 1

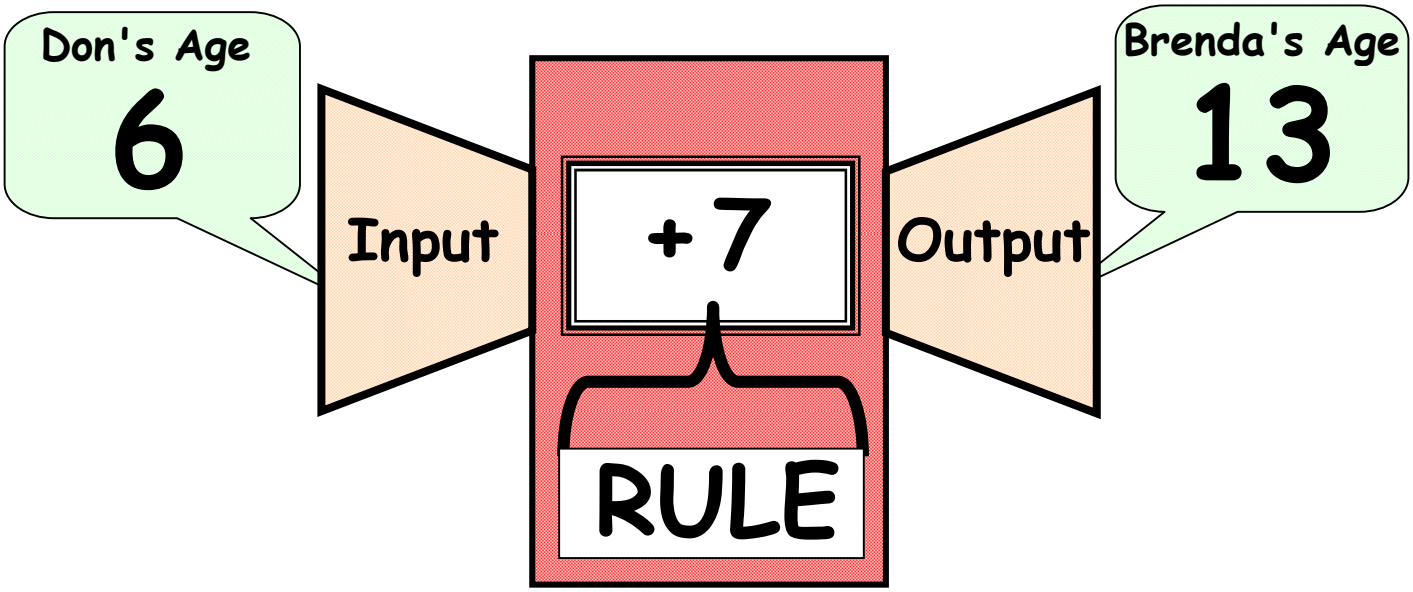
Brenda's brother Don was born on her seventh birthday. She is exactly 7 years older than Don. Her age will change and his age will change, but Brenda will always be 7 years older than Don. The **relationship** between their ages will not change.

This relationship is called a **function** because Don's age is a function of Brenda's age. There are several ways a function can be shown.

- A function can be represented with a diagram.

  A function is like a machine. When the input changes, the output will also change.





For this machine, the rule for the relationship between Don's age and Brenda's age is + 7.

When the input changes, the output will also change. But for any input there will only be one possible output.

When Don's age is 9, Brenda's age is 9 + 7, so Brenda's age is 16.

- A function can be shown using an equation. This is the rule for the function.

Don's age ← $d + 7 = b$ → Brenda's age

- A function can be shown using an input-output table.

  The **input** is the **position** in the sequence of numbers in the pattern. The input is also called the **term**.

  The **output** is the **value** of each position in the sequence. The output is also called the **value** of the **term**.

Don's age → | Input, Position | Function Rule | Output, Value | ← Brenda's age

d	***d* + 7**	***b***
0 ← First term	0 + 7	7 ← Value of term
1 ← Second term	1 + 7	8 ← Value of term
2	2 + 7	9
3	3 + 7	10
4	4 + 7	11
5	5 + 7	12

Input is Don's age. **Output** is Brenda's age.

For any input position, there is only **one** possible output value.

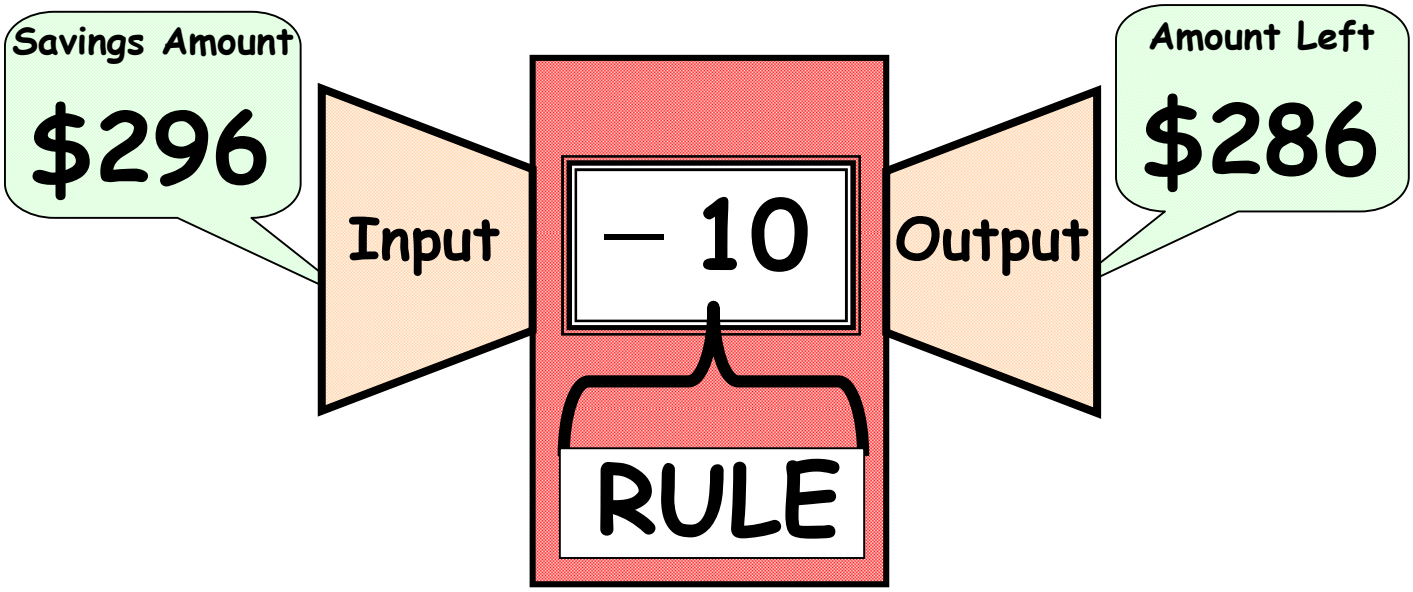
The **first term** in the pattern is 0.
The **value** of the **first term** is 7.

For any **term**, there is only **one** possible **value**.

EXAMPLE 2

Paul saved $296 this summer from mowing lawns. When school begins he plans to spend $10 of his savings each week. How much of his savings will Paul have left at the end of the first six weeks of school?

- This problem can be represented with a function machine diagram.



For this machine, the rule for the relationship between amount of savings and amount of savings left is — 10.

When savings is $296, the savings left is 296 — 10, so the savings left is $286.

- This problem can be represented by a rule.

Savings ← $s - 10 = x$ → Savings left

- This problem can be represented by an input-output table.

Input, Position	Function Rule	Output, Value
s	***s* – 10**	***x***
296	296 – 10	286
286	286 – 10	276
276	276 – 10	266
266	266 – 10	256
256	256 – 10	246
246	246 – 10	236

Savings → Input, Position; Output, Value ← Savings left

First week → 296; 286 ← Savings left

Second week → 286; 276 ← Savings left

The **Input** is the amount of savings.

The **Output** is the amount of savings left.

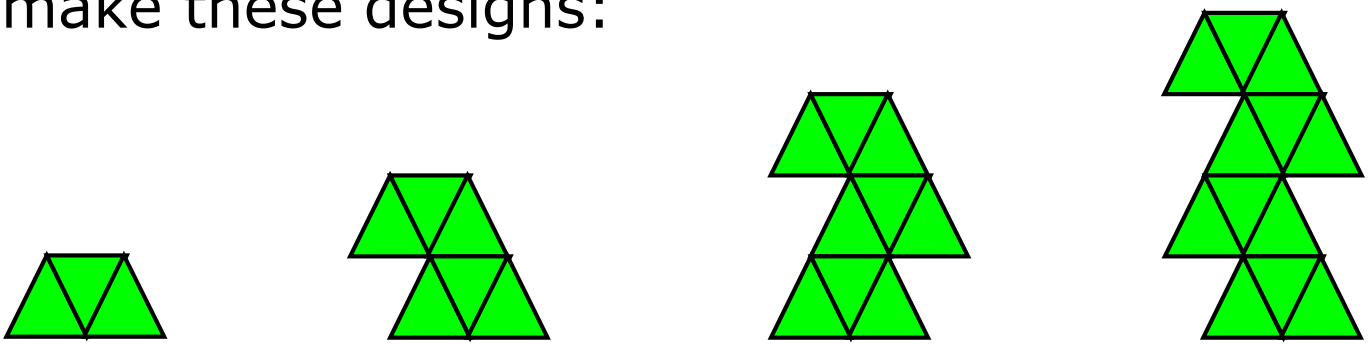
The **first term** in the pattern is 296.

The **value** of the **first term** is 286.

The **sixth term** in the pattern represents the sixth week, so Paul will have $236 of his savings left at the end of the sixth week of school.

Problem-Solving 2

Jeremy used green triangle pattern blocks to make these designs:



Design 1 Design 2 Design 3 Design 4

1. Make an input-output table and record the number of each design and the number of green pattern blocks in each design.

2. If he continues the pattern, how many green pattern blocks will he use for his 6th design? Explain your answer.

3. How many green pattern blocks will he use for his 9th design? Explain your answer.

4. What is the rule for this pattern? Explain your answer.

5. Record the sequence for the first five terms in this pattern. Explain the sequence.